STRUCTURAL-PHENOMENOLOGICAL THEORY OF THE STRESSED STATE IN AN ARBITRARILY FLOWING DILUTE SUSPENSION OF RIGID DUMBBELL-SHAPED PARTICLES IN A POWER-LAW LIQUID

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INTRODUCTION

At the present time, the rheology of dilute suspensions of rigid dumbbells in a Newtonian solvent has been studied rather completely. A review of work devoted to this problem, in which a structural approach was used in the determination of the effective viscosity in simple flows, or of the tensor of the stresses in arbitrary flows, is indicated in the bibliography of [1]. The tensor of the stresses in an arbitrarily flowing dilute suspension of rigid dumbbells in a Newtonian solvent is found using a structural-phenomenological approach [2].

The present work discusses dilute suspensions of rigid dumbbells (two rigid spheres of radius a, joined by a rigid connection) in a non-Newtonian solvent, whose rheological equation of state has the form

$$\tau_{ij} = -p_s\delta_{ji} + 2m|2d_{km}d_{mk}|^{(n-1)/2}d_{ij}, \tag{0.1}$$

where τ_{ij} is the tensor of the stresses; p_s is the pressure; δ_{ij} is a Kronecker symbol; $d_{km} = (1/2)(v_{k,m} + v_{m,k})$ is the tensor of the deformation rates; v_{ij} is the tensor of the velocity gradients; m is the index of the consistency of the liquid; and n is a parameter, characterizing the degree of non-Newtonian behavior of the liquid. With $n < 1$, this model describes pseudoplastic liquids and with $n > 1$, dilatant liquids. The case $n = 1$ corresponds to Newtonian behavior. In this case, the parameter m is the Newtonian viscosity of the liquid.

1. The Structural Theory of Viscosity in Simple Shear Flow and in Flow with Monaxial Elongation. The basic assumptions are as follows: interaction between suspended particles is neglected; hydrodynamic interaction between the ends (spheres) in dumbbell-shaped particles is not taken into consideration; the motion of the suspended particles takes place under the action only of the hydrodynamic forces of the flow (there is no effect of rotational Brownian movement or of electrical or magnetic fields); the moment of inertia of the suspended particles is neglected; the solvent interacts with the suspended particles as with hydrodynamic bodies; and the flow of the solvent around the spheres of a dumbbell is discussed in the Stokes approximation.

The flow of a homogeneous power-law liquid (0.1) around a spherical particle in the Stokes approximation was first considered in [3], the results of which were refined in [4-7]. The resistance coefficient of a sphere of radius a, moving with the velocity U, is determined by the relationship [7].

$$\zeta = 2\pi J(n)mU^{n-1}a^{2-n}; \tag{1.1}$$

in accordance with [3],

$$J(n) = 2(12/n^2)^{(n+1)/2}F(n), \tag{1.2}$$

where the function F(n) is tabulated [5]; specifically, $J(1) = 3$ (Newtonian liquid); the resistance coefficient $\zeta = 6\pi ma$.

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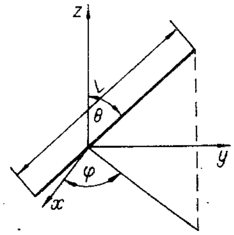

Fig. 1

The angular position of a dumbbell in space will be characterized by the angles φ, θ (see Fig. 1), with respect to a system of rectangular Cartesian coordinates (x, y, z), with an origin coinciding with the center of the axis of the particle. It is assumed that the velocity of the particle coincides with the velocity of the solvent.

The angular velocity of the dumbbell $\boldsymbol{\omega} = \{\omega_\varphi, \omega_\theta\}$ in simple shear flow

$$v_x = 0,\ v_y = Kx,\ v_z = 0,\ K = \text{const} \tag{1.3}$$

and in a flow with monaxial elongation

$$v_x = -(q/2)x,\ v_y = -(q/2)y,\ v_z = qz,\ q = \text{const} \tag{1.4}$$

of the medium under consideration is obtained by equating to zero the moment of the hydrodynamic forces $\mathbf{M} = \{M_\theta, M_\varphi\}$ acting on the particle:

$$M_\varphi^\perp = \zeta_\varphi^\perp L \sin\theta\left(\frac{KL}{2}\sin\theta\cos^2\varphi - \dot{\varphi}\frac{L}{2}\sin\theta\right),$$

$$M_\theta^\perp = \zeta_\theta^\perp L\left(\frac{KL}{8}\sin 2\varphi\sin 2\theta - \dot{\theta}\frac{L}{2}\right),$$

$$M_\varphi^\parallel = \zeta_\varphi^\parallel L\sin\theta\left(0 - \dot{\varphi}\frac{L}{2}\sin\theta\right),$$

$$M_\theta^\parallel = \zeta_\theta^\parallel L\left(\frac{L}{2}q\sin\theta\cos\theta - \dot{\theta}\frac{L}{2}\right),$$

where L is the distance between the centers of the spheres of the dumbbell; the first terms in parentheses are the components of the velocity of the liquid in the vicinity of a sphere of the dumbbell, in directions of the change in the angles φ and θ perpendicular to the axis of the particle; the second terms are the linear velocities of the end of the particle (a sphere) in the direction of the change in the angles φ and θ; the dots above φ and θ denote the total derivatives with respect to the time;

$$\zeta_\varphi^\perp = 2\pi J(n)\, ma^{2-n}\left(\frac{L}{2}\right)^{n-1}\left|\sin\theta\,(K\cos^2\varphi - \dot{\varphi})\right|^{n-1},$$

$$\zeta_\theta^\perp = 2\pi J(n)\, ma^{2-n}\left(\frac{L}{2}\right)^{n-1}\left|\frac{K}{4}\sin 2\varphi\sin 2\theta - \dot{\theta}\right|^{n-1},$$

$$\zeta_\varphi^\parallel = 2\pi J(n)\, ma^{2-n}\left(\frac{L}{2}\right)^{n-1}\left|\dot{\varphi}\sin\theta\right|^{n-1},$$

$$\zeta_\theta^\parallel = 2\pi J(n)\, ma^{2-n}\left(\frac{L}{2}\right)^{n-1}\left|q\sin\theta\cos\theta - \dot{\theta}\right|^{n-1}$$

in accordance with (1.1). The indices $\perp$ and $\parallel$, here and in what follows, denote simple shear flow and flow with nonaxial elongation.

Under these circumstances, the components of the angular velocity of the particles are determined by the equations

$$\omega_\varphi^\perp \equiv \dot{\varphi} = K\cos^2\varphi,$$
$$\omega_\theta^\perp \equiv \dot{\theta} = \frac{K}{4}\sin 2\varphi\sin 2\theta; \tag{1.5}$$

$$\omega_\varphi^{\parallel} \equiv \dot{\varphi} = 0,$$
$$\omega_\theta^{\parallel} \equiv \dot{\theta} = \frac{q}{2}\sin 2\theta, \tag{1.6}$$

which coincide with the corresponding equations for a Newtonian solvent.

From Eqs. (1.5) and (1.6) it follows that the linear velocity of the liquid in the vicinity of a sphere is perpendicular to the axis of the particle. Therefore, the relative velocity of the solvent u_0, flowing around the end of a particle, is directed along the axis of the particle. In flows (1.3), (1.4), it is determined by the relationships

$$u_0^{\perp} = \frac{L}{2} K \sin^2\theta \sin\varphi \cos\varphi; \tag{1.7}$$

$$u_0^{\parallel} = \frac{qL}{4}(2\cos^2\theta - \sin^2\theta). \tag{1.8}$$

Then the rate of dissipation of energy with the flow of a solvent around a dumbell (two spheres) has the form

$$E_0 = 2\langle \zeta_0 u_0^2 \rangle, \tag{1.9}$$

where averaging is carried out over all the angular positions of the axes of the particles, using the distribution function $F(\varphi, \theta)$, determined, as for a suspension in a Newtonian solvent, from the equation

$$\operatorname{div}(\omega F) = 0,$$

and the coefficient of the resistance ζ_0, in accordance with (1.1), (1.7), and (1.8), is determined by the relationships

$$\zeta_0^{\perp} = 2\pi J(n) m a^{2-n} \left(\frac{L}{2}\right)^{n-1} |K \sin^2\theta \sin\varphi \cos\varphi|^{n-1}; \tag{1.10}$$

$$\zeta_0^{\parallel} = 2\pi J(n) m a^{2-n} \left(\frac{L}{2}\right)^{n-1} \left|\frac{q}{2}(2\cos^2\theta - \sin^2\theta)\right|^{n-1}. \tag{1.11}$$

The rate of the dissipation of energy in unit volume of the suspension under these circumstances is defined as

$$E = E_s + N E_0, \tag{1.12}$$

where E_s is the rate of dissipation of energy in unit volume of the solvent in the absence of suspended particles; N is the number of suspended particles in unit volume of the suspension.

For the flows (1.3), (1.4), from (1.3), taking account of (1.7)-(1.11), there is obtained*

$$E^{\perp} = m|K|^{n+1} + 4\pi N J(n) m a^{2-n} \left(\frac{L}{2}\right)^{n+1} \left\langle \left|\frac{K}{2}\sin^2\theta \sin 2\varphi\right|^{n+1} \left(\frac{K}{2}\sin^2\theta \sin 2\varphi\right)^2 \right\rangle; \tag{1.13}$$

$$E^{\parallel} = m|3q^2|^{\frac{n+1}{2}} + 4\pi N J(n) m a^{2-n} \left(\frac{L}{2}\right)^{n+1} \left\langle \left|\frac{q}{2}(\cos 2\theta + \cos^2\theta)\right|^{n+1} \left(q\frac{\cos 2\theta + \cos^2\theta}{2}\right)^2 \right\rangle. \tag{1.14}$$

Relationships (1.13), (1.14) permit determining the effective viscosity μ_a of the medium under consideration in flows (1.3), (1.4).using the formulas

$$\mu_a^{\perp} = \frac{E^{\perp}}{K^2}, \quad \mu_a^{\parallel} = \frac{E^{\parallel}}{q^2}.$$

2. Structural-Phenomenological Theory. It follows from the preceding section that the stressed state in a dilute suspension of rigid dumbbells in a power-law liquid must be a function not only of the shear rate, but also of the orientation of the suspended particles. Therefore, the tensor of the stresses, characterizing the stressed state in an arbitrarily flowing dilute suspension of rigid dumbbells in an incompressible power-law liquid (0.1), in the absence of external force fields, will be sought, analogously to [8], in the form

$$t_{ij} = f_{ij}(d_{km}, n_l), \tag{2.1}$$

*As in Russian original — Publisher.

where n_l is a unit vector, characterizing the orientation of a dumbbell-shaped particle in the system of coordinates (x, y, z):

$$n_x = \cos\varphi \sin\theta,\ n_y = \sin\varphi \sin\theta,\ n_z = \cos\theta.$$

To obtain the rheological equation of state (2.1), we assume that the matrix $\{t_{ij}\}$ is an isotropic function of the matrices $\{d_{ij}\}$ and $\{n_i n_j\}$, and we use the result of Rivlin and Ericksen [9], determining the overall form of an isotropic function of a matrix of the 3rd rank from two other matrices of the 3rd rank.

Under these circumstances, we obtain

$$t_{ij} = \alpha_0\delta_{ij} + \alpha_2 n_i n_j + \alpha_3 d_{ik}d_{kj} + \alpha_4(d_{ik}n_j n_k + d_{kj}n_k n_i) + \alpha_1 d_{ij} + \alpha_5(n_i n_k d_{km}d_{mj} + n_m n_j d_{ik}d_{km}), \quad (2.2)$$

where $\alpha_i (i = 0,1, \ldots, 5)$ are functions of the invariants

$$I_2 = d_{km}d_{mk},\ I_3 = d_{mk}d_{kl}d_{lm},$$
$$J_1 = d_{km}n_k n_m,\ J_2 = d_{km}d_{ml}n_k n_l. \quad (2.3)$$

Since the Eqs. (1.5), (1.6), determining the angular velocities $\boldsymbol{\omega}^{\perp}$ and $\boldsymbol{\omega}^{\|}$, coincide with the corresponding equations for suspensions with a Newtonian solvent, the determining equation for the vector of the orientation will be sought in the form (analogously to [8])

$$\hat{n}_i = g_i(n_l,\ d_{km}), \quad (2.4)$$

where $\hat{n}_i = \dot{n}_i - \omega_{ij}n_j$, $\omega_{ij} = (1/2)\ (v_{i,j} - v_{j,i})$ is the tensor of the vorticity of the velocity. Using the Hamilton – Cayley theorem, we obtain

$$g_i = \beta_1 n_i + \beta_2 d_{ij}n_j + \beta_3 d_{ik}d_{kj}n_j,$$

where β_1, β_2, β_3 are functions of the invariants (2.3). Taking into consideration that $n_i\dot{n}_i = n_i\hat{n}_i = n_i g_i = 0$, (2.4) finally assumes the form

$$\dot{n}_i - \omega_{ij}n_j = \beta_2(d_{ij}n_j - d_{km}n_k n_m n_i) + \beta_3(d_{ik}d_{kj}n_j - d_{km}d_{mp}n_k n_p n_i). \quad (2.5)$$

We determine the parameters β_2 and β_3 comparing $\omega^{\perp}\{\omega_{\varphi}^{\perp},\ \omega_{\theta}^{\perp}\}$ and $\omega^{\|}\{\omega_{\varphi}^{\|},\ \omega_{\theta}^{\|}\}$, determined from (2.5) for the flows (1.3), (1.4) with their values from Eqs. (1.5), (1.6).

Under these circumstances, we obtain $\beta_2 \equiv 1$, $\beta_3 \equiv 0$.

Thus, the equation determining the angular velocity of the axis of the dumbbell in an arbitrary flow has the form

$$\dot{n}_i = (\omega_{ij} + d_{ij})n_j - d_{km}n_k n_m n_i. \quad (2.6)$$

Equation (2.6) coincides with the corresponding equation for a dilute suspension of rigid dumbbells in a Newtonian solvent [2].

It follows from Eq. (2.6) that, as in a Newtonian solvent [2], the axes of the particles in an arbitrary flow of a suspension are distributed nonuniformly over the angular positions. The distribution function of the axes of the particles over the angular positions is determined from the equation

$$\frac{\partial}{\partial n_i}(F\dot{n}_i) = 0. \quad (2.7)$$

As the rheological equation of state of a dilute suspension of rigid dumbbells in a power-law liquid (0.1) we take relationship (2.2), averaged using the distribution function $F(n_i)$, determined by Eqs. (2.6), (2.7):

$$T_{ij} = \langle t_{ij} \rangle = -p\delta_{ij} + \langle \alpha_1 \rangle d_{ij} + \langle \alpha_2 n_i n_j \rangle + \langle \alpha_3 \rangle d_{ik}d_{kj} +$$
$$+ \langle \alpha_4 n_j n_k \rangle d_{ik} + \langle \alpha_4 n_k n_i \rangle d_{kj} + \langle \alpha_5 n_i n_k \rangle d_{km}d_{mj} + \langle \alpha_5 n_m n_j \rangle d_{ik}d_{km}. \quad (2.8)$$

TABLE 1

$\alpha(n)$	n	$2^{1-n}J(n)$	$\alpha(n)$	n	$2^{1-n}J(n)$
155,5555	0,2	15,7786	1,5π	1,0	3
80,6823	0,3	12,4773	1,5335	1,25	1,3508
50,2122	0,4	10,5692	$0{,}2683\cdot10^{-1}$	2,0	$0{,}5365\cdot10^{-1}$
33,8003	0,5	9,1149	$0{,}1362\cdot10^{-2}$	2,5	$0{,}4408\cdot10^{-2}$
23,4209	0,6	7,7939	$0{,}7066\cdot10^{-5}$	3,33	$0{,}4804\cdot10^{-4}$
16,2591	0,7	6,5100	$0{,}8388\cdot10^{-7}$	4,0	$1{,}0064.10^{-6}$
11,1211	0,8	5,2626	$0{,}8293\cdot10^{-10}$	5,0	$0{,}2253\cdot10^{-8}$
7,4541	0,9	4,1128			

To determine the rheological parameters α_i(i = 1, ..., 5), we compare the rate of dissipation of energy in unit volume of the suspension in an arbitrary flow, obtained using (2.8):

$$E = T_{ij}d_{ij} = \langle \alpha_1 \rangle d_{ij}d_{ij} + \langle \alpha_2 n_i n_j \rangle d_{ij} + \langle \alpha_3 \rangle \times$$
$$\times d_{ik}d_{kj}d_{ij} + \langle \alpha_4 n_k n_j \rangle d_{ik}d_{ij} + \langle \alpha_4 n_k n_i \rangle d_{kj}d_{ij} +$$
$$+ \langle \alpha_5 n_i n_k \rangle d_{km}d_{mj}d_{ij} + \langle \alpha_5 n_m n_j \rangle d_{ik}d_{km}d_{ij} \tag{2.9}$$

with the rate of the dissipation of energy obtained using the structural method.

From Eq. (2.6), for an arbitrary flow, as from (1.5), (1.6) for the flows (1.3), (1.4), it follows that the relative velocity of the solvent, flowing around the end of a dumbbell-shaped particle, is directed along the particle and is equal to

$$u_0 = (L/2)d_{km}n_k n_m. \tag{2.10}$$

The rate of dissipation of energy with the flow of a solvent around a dumbbell in this case is determined using formula (1.9), where the averaging is carried out using a distribution function, obtained from Eqs. (2.6), (2.7), and the resistance coefficient ζ_0, in accordance with (1.1), (2.10), is determined by the relationship

$$\zeta_0 = 2\pi J(n) m a^{2-n} (L/2)^{n-1} |d_{km}n_k n_m|^{n-1}.$$

Thus, the rate of dissipation of energy in unit volume of a suspension in an arbitrary flow, in accordance with (1.12), has the form

$$E = 2m|2d_{km}d_{mk}|^{(n-1)/2}d_{ij}d_{ji} + 4\pi N J(n) m a^{2-n}(L/2)^{n+1} \langle |d_{km}n_k n_m|^{n-1}(d_{km}n_k n_m)^2\rangle. \tag{2.11}$$

It can be verified that, for the flows (1.3), (1.4), expression (2.11) coincides with (1.13), (1.14).

Comparing (2.9) with (2.11), we obtain

$$\alpha_1 = 2m|2I_2|^{((n-1)/2)},$$
$$\alpha_2 = N\pi J(n)2^{1-n}ma^{2-n}L^{n+1}|J_1|^{n-1}J_1,$$
$$\alpha_3 \equiv \alpha_4 \equiv \alpha_5 \equiv 0.$$

Therefore, the rheological equation of state of a dilute suspension of rigid dumbbells in an incompressible power-law liquid has the form

$$T_{ij} = -p\delta_{ij} + 2m|2d_{km}d_{mk}|^{(n-1)/2}d_{ij} + N\pi J(n)2^{1-n}ma^{2-n}L^{n+1}\langle |d_{km}n_k n_m|^{n-1}d_{km}n_k n_m n_i n_j\rangle. \tag{2.12}$$

With n = 1 (Newtonian solvent), we arrive at the rheological equation, obtained in [2]:

$$T_{ij} = -p\delta_{ij} + 2md_{ij} + N3\pi maL^2 d_{km}\langle n_k n_m n_i n_j \rangle.$$

The dependence of $2^{1-n}J(n)$ on n, taking account of (1.2) and the data of [5], is given in Table 1.

A rigid dumbbell can serve as a model of a rigid rod-shaped particle of cylindrical form, with a length L and a diameter d. Here the radius a of the spheres of the dumbbell is determined by the relationship

$$a = W(d)/3\pi m L^2, \tag{2.13}$$

where W is the coefficient of rotational friction of a rod-shaped particle of length L in a Newtonian liquid with a viscosity $\mu = m$, determined experimentally; $3\pi m a L^2$ is the coefficient of rotational friction of a dumbbell in a Newtonian liquid with a viscosity $\mu = m$.

Therefore, Eq. (2.12), in which the value of a is determined by (2.13), can serve as the rheological equation of state of dilute suspensions of rigid rod-shaped particles in a power-law liquid. In this case, the number of particles in unit volume of the suspension N can be expressed in terms of the volumetric concentration Φ of the suspended particles:

$$N = 4\Phi/L\pi d^2.$$

As an example, let us consider plane Couette flow

$$v_x = 0,\ v_y = Kx;\ K = \text{const}. \tag{2.14}$$

From (2.6) we obtain the result that the suspended particles rotate with the angular velocity

$$\dot{\varphi} = K\cos^2\varphi. \tag{2.15}$$

From (2.8) it follows that

$$\mu_a^{\frac{1}{}} = T_{xy}/K = m|K|^{n-1} + 4\pi N J(n)\, m a^{2-n} (L/2)^{n+1} \langle |K \sin\varphi \cos\varphi|^{n-1} \sin^2\varphi \cos^2\varphi \rangle. \tag{2.16}$$

The distribution function for the averaging in (2.16) is found from the equation

$$\frac{\partial}{\partial\varphi}(\dot{\varphi}F) = 0, \tag{2.17}$$

where $\dot{\varphi}$ is determined from (2.15). The solution of (2.17), satisfying the condition of normalizing

$$\frac{1}{2\pi}\int\limits_0^{2\pi} F(\varphi)\, d\varphi = 1,$$

has the form

$$F(\varphi) = (1/2\pi)1/\cos^2\varphi. \tag{2.18}$$

From (2.16), taking account of (2.18), it follows that, in the flow (2.14), the medium under consideration behaves like a power-law fluid with a degree of non-Newtonian behavior of the solvent and with the effective consistency

$$m_a = m\left[1 + 2NJ(n)\, a^{2-n}\left(\frac{L}{2}\right)^{n+1}\int\limits_0^{2\pi} |\sin\varphi\cos\varphi|^{n-1}\sin^2\varphi\, d\varphi\right].$$

Using the fact that [10]

$$\int\limits_0^{\frac{\pi}{2}} \sin^{n+1}\varphi \cos^{n-1}\varphi\, d\varphi = \frac{\Gamma\left(\frac{n}{2}+1\right)\Gamma\left(\frac{n}{2}\right)}{2\Gamma(n+1)},$$

and taking account of (1.2), we obtain

$$m_a = m[1 + 4Na^{2-n}L^{n+1}(3/n^2)^{(n+1)/2}F(n)\Gamma^2(n/2)/\Gamma(n)].$$

The dependence of

$$[(m_a - m)/Nm]1/a^{2-n}L^{n+1} = \alpha(n)$$

on n is given in Table 1. With n = 1 (Newtonian solvent)

$$[(m_a - m)/Nm]1/aL^2 = (3/2)\pi,$$

which coincides with the results of [2].

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HEAT TRANSFER WITH THE FLOW OF STRUCTURALLY VISCOUS MEDIA IN TUBES AND CHANNELS

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UDC 536.25

In various branches of modern technology, wide use is made of so-called structurally viscous media, which, in their physical properties, differ considerably from ordinary Newtonian liquids. Structurally viscous media include high-polymer, colloidal, bulk, coarsely dispersed, and other systems, for which the Newton hypothesis of a linear dependence between the rate of deformation and the stress no longer holds. A nonlinear dependence between the stress and the gradient of the rate of flow is the most characteristic special feature of non-Newtonian liquids [1]; this dependence is frequently expressed by the Ostwald formula

$$\tau = k(dw/dr)^m. \quad (1)$$

For a laminar, hydrodynamically stabilized flow of anomalous liquids with an exponential rheological law (1), the field of the velocities in a round tube and a plane-parallel channel is expressed by the formula

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